7. V. G. Babskii, N. D. Kopachevskii, A. D. Myshkis, et al., Hydromechanics of Weightlessness [in Russian], Nauka, Moscow (1976).
8. V. Blyashke, Differential Geometry [in Russian], ONTI, Moscow—Leningrad (1935).
9. V. V. Rumyantsev, "Theory of motion of solids with cavities filled with a fluid," Prikl. Mat. Mekh., 30, No. 1 (1966).
10. A. D. Tyuptsov, "Hydrostatics and weak force fields. Stability of equilibrium forms of the fluid surface," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2 (1966).
11. S. Chandrasekhar, Ellipsoidal Figures of Equilibrium, Yale Univ. Press, CT (1970).
12. V. V. Rumyantsev and A. S. Ozaraner, Stability and Stabilization of Motion in Relation to Some Variables [in Russian], Nauka, Moscow (1987).
13. L. V. Ovsyannikov, N. I. Makarenko, V. I. Nalimov, et al., Nonlinear Problems of the Theory of Internal and Surface Waves [in Russian], Nauka, Novosibirsk (1985).
14. V. A. Samsonov, "Stability and bifurcation of the equilibrium of a body with fluid," Nauchn. Tr. Inst. Mekh. Mosk. Gos. Univ., No. 16 (1971).
15. V. A. Vladimirov, "Analogs of the Lagrange theorem in the hydrodynamics of turbulent and stratified fluids," Prikl. Mat. Mekh., 50, No. 5 (1986).
16. A. N. Kolmogorov and S. V. Fomin, Elements of the Theory of Functions and Functional Analysis [in Russian], Nauka, Moscow (1972).
17. L. D. Landau and E. M. Lifschitz, Mechanics [in Russian], Nauka, Moscow (1965).
18. R. Fjortoft, "On the frontogenesis and cyclogenesis in the atmosphere, Pt. I. On the stability of the stationary circular vortex," Geophys. Publ., 16, No. 5 (1944).
19. R. Fjortoft, "Application of integral theorems in deriving criteria of stability for laminar flows and for the baroclinic circular vortex," Geophys. Publ., 17, No. 6 (1950).
20. A. Eliassen and E. Kleinschmidt, "Dynamic meteorology," Handbuch Phys., 48, (1957).
21. K. Ooyama, "On the stability of the baroclinic circular vortex: a sufficient criterion for instability," J. Atmos. Sci., 23, No. 1 (1966).
22. J. Lighthill, Waves in Fluids [Russian translation], Mir, Moscow (1981).
23. L. I. Sedov, Continuum Mechanics [in Russian], Vol. 1, Nauka, Moscow (1973).
24. L. S. Srubshchik and V. I. Yudovich, "Note on the stability of membrane solutions in the nonlinear theory of plates and shells," Prikl. Mat. Mekh., 30, No. 1 (1966).

DEVELOPMENT OF VISCOSITY INSTABILITY IN A POROUS MEDIUM

O. B. Bocharov and V. V. Kuznetsov

UDC 532.546

The interest in the problem of the stability of two-phase flows undergoing filtration is due mainly to the problem of maximizing the recovery of oil from underground when water or other agents which are immiscible with oil are pumped into the reservoir. When the ratio of the viscosities is large, the displacement of hydrocarbon liquids by water in a porous medium is essentially an unstable process. Instability of the displacement front leads to the formation of "tongues" of liquid which increase in size over time. The linear analysis of stability for piston-like displacement performed in [1] showed that the increase in the amplitude of the tongues is exponential in character. In [2], stability within the framework of a linear approximation was analyzed for the Muskat—Leverett model with allowance for the erosion of the displacement front due to capillary forces. The growth of tongues after loss of stability was analyzed numerically without allowance for capillary forces in [3] for uniform porous media and in [4] for microscopically nonuniform porous media. A detailed analysis of studies of viscosity instability in porous media was given in [5]. At the same time, there has been little study of the stage of nonlinear tongue growth with allowance for the two-phase character of flow behind the displacement front. Here, within the framework of the Baclay—Leverett model, i.e., without allowance for capillary forces, we numerically study the structure of the flow region behind the displacement front in the unstable regime at the nonlinear stage of tongue growth.

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 116-120, March-April, 1989. Original article submitted June 9, 1987; revision submitted December 8, 1987.

We will examine the motion of two immiscible incompressible fluids in a uniform horizontal bed. A two-dimensional formulation will be used. The system of equations for determining the pressure and the saturation of the displacing phase s has the following form in dimensionless variables, with no allowance being made for the capillary pressure discontinuity between the phases and with the assumption of a linear filtration law [6]

$$\partial s/\partial t + \operatorname{div}(F(s)\mathbf{v}) = 0,\ \mathbf{v} = -M(s)\operatorname{grad} p,\ \operatorname{div}\mathbf{v} = 0. \tag{1}$$

Here and below, $F(s) = k_1(s)/[k_1(s) + \mu k_2(s)]$ is the Baclay–Leverett function; $M(s) = [k_1(s)\mu^{-1} + k_2(s)]/k_2(0)$ is the total mobility of the phases with saturation s, referred to the initial mobility with s = 0; $k_1(s)$ and $k_2(s)$ are the relative phase permeabilities; $\mu = \mu_1/\mu_2$ is the ratio of the viscosities of the fluids; v is the total filtration rate, made dimensionless with respect to the rate of filtration of the displacing fluid v_0 at x = 0; p is pressure, made dimensionless with respect to $p_0 = v_0\mu_2\lambda/kk_2(0)$; t is time, made dimensionless with respect to $t_0 = v_0/m\lambda$; x and y are the space coordinates, made dimensionless with respect to the wavelength of the perturbation λ; m is porosity; k is the absolute permeability of the porous medium; ℓ is the length of the theoretical region; the subscripts 1 and 2 denote the displacing fluid and the fluid being displaced, respectively.

Flow is studied in the rectangular region $x \in [0, a]$, $y \in [0, 0.5]$ with impermeable boundaries at y = 0, y = 0.5, which models the symmetry of the flow. The study is conducted with a specified pressure p = 0 for $x = a$, $a = \ell/\lambda$ and specified values of the rates of flow of the fluids for x = 0. At the initial moment of time, the entire region is filled by the fluid being displaced (s = 0).

For a difference approximation of the above-formulated boundary-value problem in the region being examined, we introduce a uniform block-centered grid with the step $h_x = h_y = h$. The solution is obtained by the IMPES method [7]. We used an explicit difference scheme of the Todd type [7] with arithmetic mean approximation of the function F(s) at half-integral points in the neighborhood of the front in order to obtain second-order accuracy in the region as a whole. The calculations were performed with a = 5, 10, and 20, which allowed us to study the asymptotic stage of development of the tongues. We used the method proposed in [3] to form the perturbations. The rate of flow of the displacing fluid at the inlet was assigned in the form of a function of time

$$v_{1,x}(0, y, t) = \begin{cases} 1 - \alpha\cos(2\pi y), & 0 \leqslant t \leqslant t_*, \\ 1, & t_* < t, \end{cases}$$
$$v_{2,x}(0, y, t) = 0, \qquad 0 \leqslant t,$$

where α determines the amplitude of the perturbation of the displacement front at the moment of time t_*. The distribution of saturation at the moment of time t_*, being the numerical solution of system (1), was regarded as the initial condition for s(x, y) at $t = t_*$. The calculations were performed for relative phase permeabilities of the form $k_1(s) = s^2$, $k_2(s) = (1 - s)^2$.

Figure 1 shows the results of calculation of the saturation fields of the pumped fluid performed at a = 10, h = 0.1, and μ = 0.1. The solid and dashed lines show forms of isolines of constant saturation of the displacing phase for different moments of time. The saturation isoline $s_c = 0.3$ corresponds to the frontal saturation at μ = 0.1 in a unidimensional Baclay–Leverett problem and describes the form of the tongue of displacing fluid. The calculations showed that the maximum rate of growth of amplitude is seen for the isoline with saturation equal to the frontal saturation (Fig. 2). The amplitude of the isoline A_s, with saturation s, was calculated as half of the projection of the isoline on the horizontal axis x, conforming to the amplitude of the isoline A_c with saturation s_c. At the nonlinear stage of tongue growth, with A > 0.1, the rate of increase in the amplitudes of the isolines $W_s = dA_s/ds$ is constant and does not depend on the amplitude of the tongue. Here, the isolines with $s > s_c$ are associated with a considerably lower growth rate. The constancy of the rate of growth of tongue amplitude in dimensionless variables indicates that, in dimensional variables, it is independent of the wavelength of the perturbation λ.

Calculations performed for μ = 0.167-0.0208 show that, regardless of the viscosity ratio, in the asymptotic stage of tongue growth the width of the tongues is equal to half the wavelength of the initial perturbation (see Fig. 1). The same is true of tongues in a piston formulation, such as in the Hele–Show model [1]. However, in contrast to this model, in a porous medium the distortion of the isoline fields is quite different for different $s > s_c$.

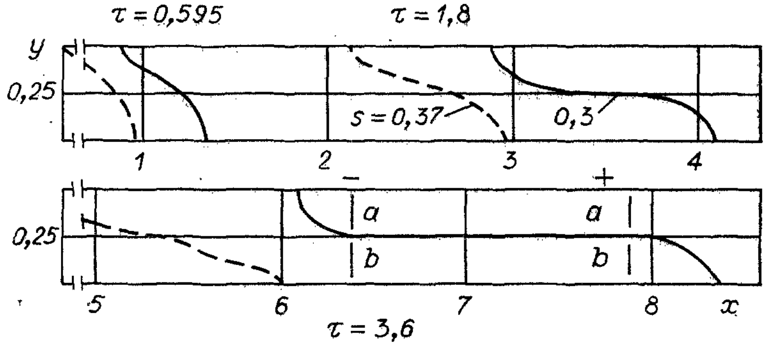

Fig. 1

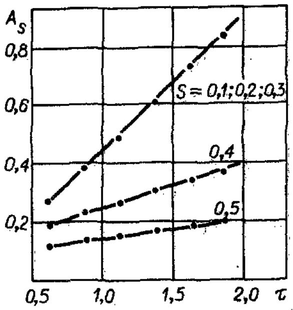

Fig. 2

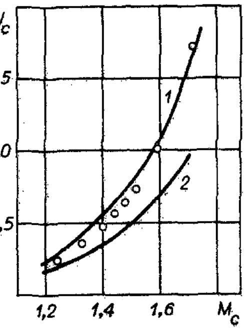

Fig. 3

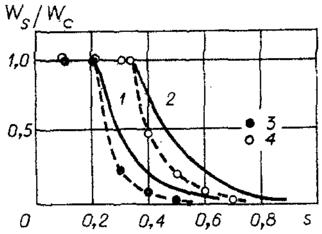

Fig. 4

It also follows from the calculations that for different moments of time (and tongue amplitudes), the saturation on the axis of a tongue at the level of its rear part s_- is roughly constant and at $\mu = 0.1$ is equal to 0.37. This pattern was also seen in calculations performed with other viscosity ratios, which shows that s_- is determined by the viscosity ratio of the fluids.

Transverse pressure gradients in the region of a tongue can be ignored for large tongue amplitudes, since the width of the tongue is constant (see Fig. 1). In accordance with the method in [1], this makes it possible to determine the rate of growth of a tongue with allowance for the two-phase character of flow behind the displacement front. We will examine flow near the front part of the tongue in the section + and near its rear part in the section −. In these sections, saturation $s = 0$ in zone a, while two-phase flow exists in zone b and the longitudinal components of total filtration velocity are determined as

$$v_+^a = -M(0)\,\partial p_+/\partial x,\quad v_+^b = -M(s_c)\,\partial p_+/\partial x;$$
$$v_-^a = -M(0)\,\partial p_-/\partial x,\quad v_-^b = -M(s_c)\,\partial p_-/\partial x, \tag{2}$$

where $\partial p_+/\partial x$ and $\partial p_-/\partial x$ are the pressure gradients in the longitudinal direction near the front and rear parts of the tongue. Here, we assume that s_- is the same for different moments of time. Equations (2) lead to the following relation for the total filtration velocities in zones a and b:

$$v_+^a/v_+^b = M(0)/M(s_c),\quad v_-^a/v_-^b = M(0)/M(s_-). \tag{3}$$

Equations (3) and the condition of constancy of the total rate of flow over the entire section

$$(1-\varphi)\,v_+^a + \varphi v_+^b = (1-\varphi)\,v_-^a + \varphi v_-^b = v_0 \tag{4}$$

(φ being the fraction of the section occupied by the tonge) determine the filtration velocities in regions a and b with allowance for the constancy of s_c and s_- for different tongue amplitudes.

In the absence of transverse flows, the rates of transport of frontal saturation in the leading and trailing parts of the tongue are found from the solution of the unidimensional Baclay–Leverett problem [6]:

$$W_+(s_c) = v_+^b F'(s_c)/m, \quad W_-(s_c) = v_-^a F'(s_c)/m. \tag{5}$$

With allowance for (3)-(5), the rate of growth of tongue amplitude $W_c = [W_+(s_c) - W_-(s_c)]/2$ has the form

$$W_c = \frac{v_0 F'(s_c)}{2m}\left[\frac{M_c}{\varphi M_c + (1-\varphi)} - \frac{1}{\varphi M_- - (1-\varphi)}\right] \tag{6}$$

$[M_c = M(s_c)$ and $M_- = M(s_-)]$. The value of s_- can also be found from Eqs. (2) and (3) by using the assumption of the absence of overflows inside the tongue. In this case, the rate of transport of saturation s_- in zone b at the level of the trailing part of the tongue $W^b(s_-) = v_-^b F'(s_-)/m$, $F'(s_-) = (dF/ds)_{s=s_-}$. The value of s_- will not change at different stages of tongue growth under the condition $W^b(s_-) = W_-(s_c)$, which leads to the following relation for s_-:

$$F'(s_-) = F'(s_c)[\varphi M_c + (1-\varphi)]/[(1-\varphi) + \varphi M_-]M_c. \tag{7}$$

Equations (6) and (7) give the rate of growth of tongues at $A_c > 0.1$. As in the numerical calculations, the growth rate in dimensional variables is independent of λ.

Figure 3 shows results of numerical calculations of the rate of growth of the amplitudes of tongues in relation to the ratio of the mobilities M_c at the displacement front (points). Also shown are the results of calculations performed with Eqs. (6) and (7) at $\varphi = 0.5$ (line 1). Line 2 represents the results of calculations with (6) performed without allowance for the change in saturation along the tongue ($s_- = s_c$). For the specified form of relative phase permeabilities, the value of M_c is determined by the ratio of the viscosities of the fluids. The calculations with (6) and (7) satisfactorily generalize the data from numerical calculations with the complete model throughout the investigated range of mobility ratios M_c.

For $s > s_c$, the rate of growth of the amplitudes of the saturation isolines depends on s (Fig. 4). Here, we present results of numerical calculations of the rate of growth of the amplitudes of isolines with saturation s, referred to the rate of growth of the amplitude of the tongues W_c, as a function of s at $\mu = 0.125$ and 0.0417 (lines 1 and 2). Tongue growth results in a redistribution of the rate of flow of the displacing fluid across the section behind the displacement front, in accordance with (2)-(4). The presumption of the absence of transverse flows behind the front for isolines with $s > s_c$ and the use of Eqs. (3) and (5) and expressions similar to (5) and (6) for the rate of shifting of the isoline s at y = 0 and y = 0.5 establish the rate of growth of isoline amplitude corresponding to the saturation s:

$$W_s = W_c F'(s)/F'(s_c). \tag{8}$$

Figure 4 shows results of calculations with (8) for the viscosity ratio $\mu = 0.125$ and 0.0417 (lines 1 and 2). Also shown are results from the complete model (points 3 and 4). The numerical results give lower values of growth rate than Eqs. (6)-(8), which is evidence of transverse flow behind the displacement front. Such flow decreases the nonuniformity of the rate of flow of the displacing fluid that forms the growing tongue.

Thus, numerical calculations have shown that during the nonlinear stage of perturbation growth, the rate of increase in the amplitude of the tongues is constant and independent of the width of the tongues and is determined by the ratio of mobilities at the displacement front. Data were obtained on the rates of growth of isolines corresponding to different saturations of the displacing fluid for different viscosity ratios μ. An approximate relation was proposed to determine the rate of growth of tongue amplitude.

LITERATURE CITED

1. P. G. Saffman and G. Taylor, "The penetration of a fluid into a porous medium or Hele-Show cell containing a more viscous liquid," Proc. R. Soc. Ser. A, 245 (1958).
2. V. M. Ryzhik and B. E. Kisilenko, "Study of the stability of the advance of an oil-water boundary in a porous medium," in: Physico-Geological Factors in the Development of Oil and Oil–Gas-Condensate Fluids [in Russian], Nedra, Moscow (1969).
3. V. M. Entov and V. B. Taranchuk, "Numerical modeling of the unstable displacement of oil by water," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5 (1979).

4. P. V. Indel'man, R. M. Kats, and M. I. Shvidler, "Numerical modeling of unstable filtrative displacement," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2 (1979).
5. R. M. Kats and V. B. Taranchuk, "Survey of studies of the stability of filtration flows," in: Dynamics of Multiphase Media [in Russian], Inst. Teor. Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1981).
6. G. I. Barenblatt, V. M. Entov, and V. M. Ryzhik, Theory of Nonsteady Filtration of Liquid and Gas [in Russian], Nedra, Moscow (1972).
7. H. Aziz and E. Settari, Mathematical Modeling of Stratified Systems [in Russian], Nedra, Moscow (1982).

DYNAMICS OF EXPLOSIVE LOADING FOR A FINITE VOLUME OF A DENSE TWO-PHASE MIXTURE

I. G. Getts and V. K. Kedrinskii

UDC 532.528+539.375

This work is devoted to the problem of breakdown of a finite volume of liquid with explosive loading concentrated at its center. It is often assumed [1] that this type of process in liquids is identical to phenomena occurring in solid bodies from the point of view of their final effect, i.e., formation of spalled layers close to the free surface with reflection from it of a strong shock wave (SW). By analogy the concept is introduced of critical tensile stresses which are accommodated by the material and which when exceeded lead to formation, for example, in the case of plane shock waves, of plane separation surfaces. In [2], on the basis of analyzing work for studying critical stresses, it was shown that experimental data often differing by an order of magnitude may be explained by the nature of loading if a liquid which always contains microinhomogeneities in the form of free gas microbubbles is considered as a two-phase material and an appropriate mathematical model is applied to it. However, as noted in [3], this approach is inadequate in order to describe the breakdown process. It is also shown there that behind a propagating rarefaction wave front there is intense development of bubble cavitation. This type of volumetric cavitation boiling embraces a significant part of the liquid, the medium becomes optically opaque, and, as can be seen from calculations, it retains hardly any tensile stresses which relax in a time of the order of 1 μsec. Nonetheless, cases are possible when within the volume of a cavitating liquid conditions are created leading to occurrence of spalling phenomena [3]. The explicit cavitation (frothy) structure of these layers only underlines the indeterminate nature of the mechanism of their formation.

The main features of the breakdown process for a finite volume of liquid with a free surface under explosive loading may be described as follows. Reflection of a strong SW from a free surface leads to formation of an unloading wave behind the front of which intense development of bubble cavitation is observed at nuclei whose role is played by microinhomogeneities: their density is of the order of 10^5-10^6 cm^{-3} [4], i.e., the process of damage initiation typical for brittle fracture dynamics [5, 6] is absent in a liquid in view of the features of its original structure. Unlimited development of cavitation bubbles leads to formation in the "boiling" liquid of a foam structure [7]. The latter, during inertial expansion, is finally transformed into a gas-droplet structure. Naturally, in each specific case, the duration of this or another stage of the breakdown process may be different and it depends markedly on loading dynamics. Nonetheless, on the basis of already known experimental and numerical studies (e.g., [2, 5, 8]) it is possible to note these typical times for the process: of the order of a microsecond for relaxation of tensile stresses, tens of microseconds for development of a cavitation zone (cavitation cluster), hundreds of microseconds for formation of a foam structure, and of the order of milliseconds for its breakdown into liquid fragments.

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 120-125, March-April, 1989. Original article submitted July 15, 1988.

